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FORMATION OF A DEVELOPED TURBULENT FLOW IN A QUADRATIC CHANNEL

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We present results of experimental investigations of aerodynamic and statistical characteristics of the region of natural transition and fully developed turbulent flow in quadratic channels.

An important place in the study of flows in pipes and channels has been the problem of the length of the entry segment in which the formation of the hydrodynamically stable state takes place. The practical application of the entry segment is most important in shorter pipes. The study of the entry segment of a turbulent flow is also important for the general understanding of the mechanism of turbulence formation.

For a long time, conflicting views were held about the length of the entry segment in pipes and channels during the turbulent flow. It was assumed here that the transition from the laminar to turbulent flow is practically sudden [1-3]. Principally, new results were obtained by Rott who showed that in the case of a minimum Reynolds numbers, the pipe contains an extended transition region with an intermittent flow regime [4]. The intermittence of the flow was explained by the fact that in the entry segment of the pipe, turbulent locks appears

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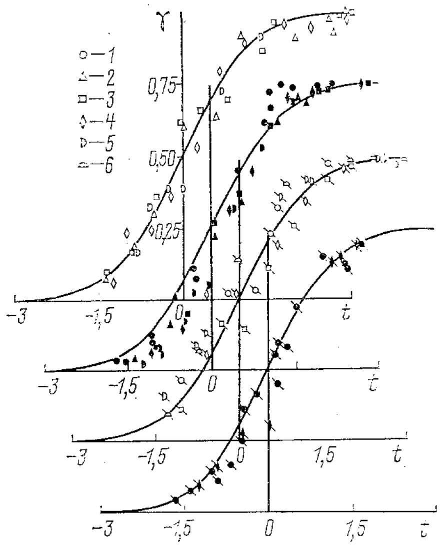

Fig. 1. Comparison of the experimental dependence on the intermittency coefficient γ on $t = (Re - \overline{Re})/\sigma$ and the normal distribution for different levels of perturbation of the flow. Points 1 correspond to x = 54, 2 to x = 46, 3 to x = 38, 4 to x = 32, 5 to x = 22, and points 6 correspond to x = 18. The empty points correspond to $Re_* = 2.25 \cdot 10^3$, full points correspond to $Re_* = 4.15 \cdot 10^3$, empty points with a stroke to $Re_* = 6.93 \cdot 10^3$, and the dark points with a stroke correspond to $Re_* = 10.89 \cdot 10^3$.

which grow along the length of the entry segment. At the end of the transition region, the growth of the turbulent locks leads to the complete disappearance of the intermediate laminar zones, and to the formation of a developed turbulent flow. As the Reynolds number increases, the position of the end of the transition region is displaced towards the entrance of the pipe, and the length of the entry segment decreases. According to the earlier notion about the development of turbulence in pipes, one should expect that the length of the entry segment increases with increasing Reynolds numbers. At critical Reynolds numbers larger than their minimum value, it has not been possible to achieve the necessary reproducibility of the experimental data about the transition even after the work of Rott [5]. The inconsistency of the experimental data, and the absence of the necessary completeness of the evidence on the experimental conditions made it impossible to form a reasonable model of the development of the flow in the entry segment of the pipe. Until recently, information about the development of turbulent flow in quadratic channels has been completely absent, although this is necessary for the solution of many technological problems, e.g., the design of hydrodynamic and aerodynamic pipes, and of special heat exchangers.

With the aim of extending the knowledge about the mechanism of natural formation of turbulence, about the characteristics of the transition region, and about the turbulent flow in quadratic channels, we begun in the mid 1960s the work with air, and subsequently water, flows. The investigations on the formation of turbulence and on the statistical characteristics of the region of natural transition were carried out by means of the scattered-light method [6, 7], using the hydrodynamical apparatus GU-2 in a channel of cross section 25 × 25 mm, with variable conditions at the entrance. The average hydrodynamic quantities of the transition region and of the developed turbulent flows were obtained on the apparatus V-1 and GU-1 with channels of cross sections 40 × 40 mm. The velocity profiles were obtained by means of microtubes of total pressure with slit entry apertures, and by means of transversing probes with scale limit of 0.01 mm. The measurements of the pressure were carried out by micromanometers. The detailed description of the experimental equipment and measuring techniques can be found in [8-10].

The first experiments showed that for all values of the Reynolds number Re, the fully developed turbulent flow is always accompanied by a transition region with an intermittent flow regime, caused by the presence of turbulent locks.

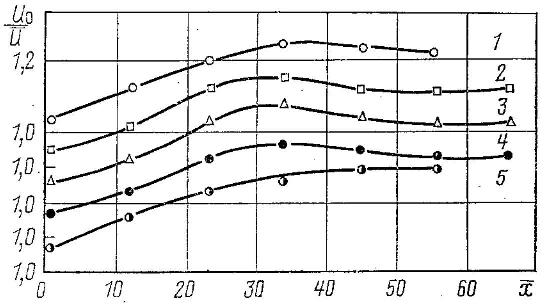

Fig. 2. Change of the magnitude of the dimensionless velocity along the hydraulic axis of the channel. Curve 1) $Re \cdot 10^{-3} = 65$; 2) $Re \cdot 10^{-3} = 40$; 3) $Re \cdot 10^{-3} = 30$; 4) $Re \cdot 10^{-3} = 20$; 5) $Re \cdot 10^{-3} = 15$.

Depending on the level of perturbation of the entry flow, the development of turbulence can take place by two ways [10]. In the case of a sufficiently high level of perturbations which ensure the transition from a laminar to turbulent flow at the minimum critical Reynolds number Re, turbulent locks are formed in the central part of the channel cross section, starting from the distance of 30 calibers from the entrance, after a considerable previous relaminarization. At low level of perturbations, turbulence is formed in a boundary laminar layer of the entry segment in the form of turbulent spots which are carried downstream, and develop into locks. The formation of turbulent spots is the concluding stage of the development of proper oscillations in the flow.

The turbulent locks appear at random along the length of the transition region. Their velocity of their edges remains unchanged along the length of the channel. For Reynolds numbers Re somewhat larger than the minimum critical value, the relative velocity of the leading edges initially increases relatively rapidly with the increasing number Re. In the region of numbers $Re \approx 5 \cdot 10^3 - 6 \cdot 10^3$, it reaches its maximum value on the order of 1.8, and decreases smoothly thereafter. The relative velocity of the trailing edges of the turbulent locks decreases monotonically with increasing Re.

The investigations of the statistical characteristics of the transition region showed that dependence of the intermittency coefficient γ and of the frequency of passage of the locks n on the Reynolds number Re can be represented by the distribution function and the probability density of the normal distribution, respectively:

$$\gamma(\mathrm{Re}) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^{\mathrm{Re}} \exp\left[-\frac{(\mathrm{Re}-\overline{\mathrm{Re}})^2}{2\sigma^2}\right] d\,\mathrm{Re}, \tag{1}$$

$$n(\mathrm{Re}) = \frac{k}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(\mathrm{Re}-\overline{\mathrm{Re}})^2}{2\sigma^2}\right]. \tag{2}$$

The parameters $\overline{Re}$ and σ in expressions (1) and (2) characterize the center and scatter of the distributions, and were determined from the experimental dependences $\gamma = f(Re)$ constructed on the probability paper [11]. The values of the coefficient k in (2) were found by the least-square method. For a constant level of perturbations of the flow advancing in the channel, the changes of the parameters $\overline{Re}$ and σ with the distance of a chosen cross section from the entrance of the channel were, for $x \lesssim 20$, described by the equations

$$\overline{\mathrm{Re}} = a + b/x, \tag{3}$$

$$\sigma = A + B/x. \tag{4}$$

The level of perturbations was determined indirectly by the magnitude of the number Re_* at which the turbulent flow in the cross section $\bar{x} = 50$ consisted of 50% of the observation time. The observation time was always chosen sufficiently long not to affect the results of measurements. The experimentally established relationship between the coefficients of Eqs. (3) and (4) with the number Re_* have the form

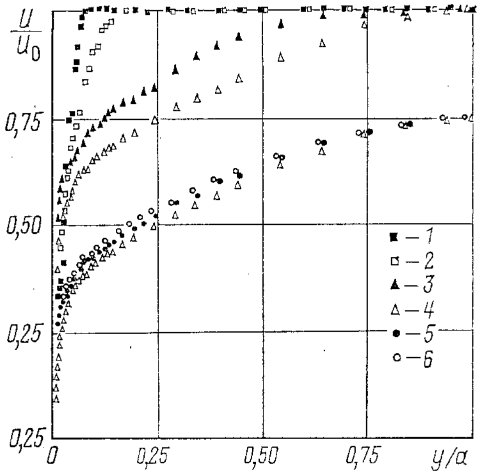

Fig. 3. Development of the turbulent velocity profile in the case of a smooth entrance of the channel. Curve 1) $\overline{x} = 0.5$; 2) $\overline{x} = 12$; 3) $\overline{x} = 23$; 4) $\overline{x} = 34$; 5) $\overline{x} = 45$; 6) $\overline{x} = 56$.

$$a = 0.20\cdot 10^3 + 0.75\,\mathrm{Re}_*,\quad A = -0.332\cdot 10^3 + 0.135\,\mathrm{Re}_*,$$
$$b = -10.06\cdot 10^3 + 12.38\,\mathrm{Re}_*,\quad B = 16.83\cdot 10^3 - 1.50\,\mathrm{Re}_*. \tag{5}$$

The comparison of the results of measurements with the normal distribution function is shown in Fig. 1. The values of the parameters $\overline{\mathrm{Re}}$ and σ were calculated from Eqs. (3), (4), and (5). It is seen that the agreement of the experimental points with the Gaussian curve is satisfactory. The deviation of the majority of points from the curve is less than 0.1γ.

The obtained analytical expressions make it possible to calculate the characteristics of the transition. For example, for the minimum Reynolds number Re ($\mathrm{Re}_* = 2.25\cdot 10^3$) the change of the intermittency coefficient along the length of the transition region is described by the formula

$$\gamma(\mathrm{Re},\ \overline{x}) \approx \frac{\overline{x}}{13.46\cdot 10^3\sqrt{2\pi}}\int_0^{\mathrm{Re}} \exp\left\{-\frac{[\mathrm{Re}\,\overline{x} - (17.8 + 1.89\,\overline{x})\cdot 10^3]^2}{3.62\cdot 10^8}\right\} d\mathrm{Re}. \tag{6}$$

It follows from (6) that for $\mathrm{Re}_0 = 1.89\cdot 10^3$, the intermittency coefficient is approximately equal to 0.1, and does not change along the length of the channel. For numbers $\mathrm{Re} < \mathrm{Re}_0$, the quantity γ decreases (the flow laminarizes), and for $\mathrm{Re} > \mathrm{Re}_0$, the value of γ increases with increasing distance from the entrance ($\overline{x} \gtrsim 30$). The larger the deviation of the number Re from Re_0, the more pronounced is the variation of γ with $\overline{x}$.

The measurements of the averaged frequency of passage of the turbulent locks showed that, in the case of transition of the first type, the magnitude of the normalizing factor $k = 0.49$ is independent of the cross section of the channel. For transitions of the second type, we have $k \approx 24\overline{x}^{-1}$. The average size of the turbulent locks increases monotonically along the length of the channel up to the end of the transition region.

The investigations of the average characteristics of the flow along the length of the transition region were carried out in the aerodynamic apparatus V-1 which ensured the least perturbation of the entry flow. The laminar flow regime remains along the whole length of the channel up to $\mathrm{Re} \approx 18\cdot 10^3$. A further increase of the Reynolds number led to the appearance of turbulence at the end of the channel. The change of the velocity of the flow along the channel axis for various Re is shown in Fig. 2. Purely laminar flow regime is characterized by a monotonic increase of the velocity along the axis up to the value $2.097\overline{U}$ [9]. The transition from the laminar to turbulent flow leads to an increase of the rate of growth of velocity. In contrast with the laminar regime, the development of the turbulent flow takes place nonmonotonically. Soon after the disappearance of the flow core, the velocity at the channel axis U_0 reaches a maximum value U_0^* ($U_0^*/\overline{U} = 1.25$), and therefore it slowly decreases approximately up to the end of the transition region. After the initial formation

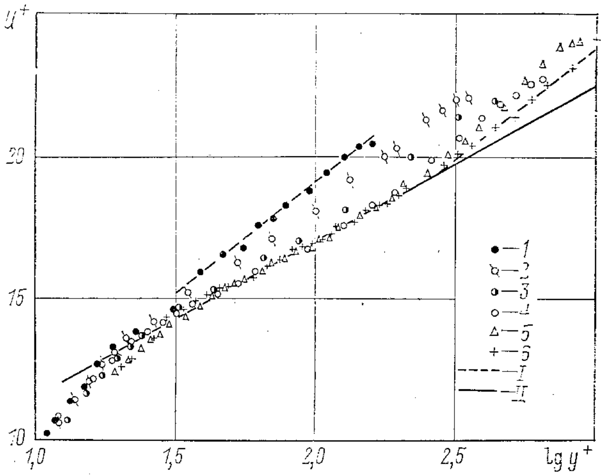

Fig. 4. Wall law for the turbulent flow. Hydrodynamic apparatus GU-1: $Re \cdot 10^{-3} = 4.8$ (curve 1); $Re \cdot 10^{-3} = 12.5$ (curve 2); $Re \cdot 10^{-3} = 15.7$ (curve 3); $Re \cdot 10^{-3} = 24.4$ (curve 4). Aerodynamic apparatus V-1 $Re \cdot 10^{-3} = 35$ (curve 5); 6) 65; I(8) — $U^+ = A + 7.8 \log (y^+)$; II (9) — $U^+ = 6.0 + 5.5 \log (y^+)$.

of turbulence at the end of the channel, the transition region moves relatively fast towards the entrance as Re increases. At the entrance of the channel, the position of the transition region noticable stabilizes. The position of U_0^* even starts to move downstream somewhat for a further considerable increase of Re. For example, in the conditions of the present work, its increase from $20 \cdot 10^3$ to $65 \cdot 10^3$ led to the displacement of the position of U_0^* from the cross section $\bar{x} \approx 30$ to the cross section $\bar{x} \approx 35$, respectively. The fall of the velocity U_0 was observed up to cross section $\bar{x} \approx 55$; after this, a new but very weak increase of the velocity U_0 was observed. The development of turbulence takes place in a similar manner in pipes of circular cross section [12].

In the entry segment of the channel the dependence $dp/d\bar{x} = f(\bar{x})$ sharply changes and has two pronounced minima. As Re increases, the position of the minimum closest to the entrance is displaced toward the entrance, and its depth decreases. The depth of the second minimum decreases. After the second minimum ($\bar{x} \approx 35$) the dependence $dp/d\bar{x} = f(\bar{x})$ reaches a plateau ($\bar{x} \gtrsim 50$) whose height is determined by the hydrodynamic resistance of the channel.

The measurement of the velocity profile in the cross sections $\bar{x} = 0.5$; 12; 23; 34; 45; 56; for Reynolds numbers $Re \cdot 10^{-3} = 10$; 15; 20; 30; 35; 40; 50; 60; 65 showed that as the velocity U_0 along the axis changes, the velocities across the whole cross section of the channel are redistributed. An example of the changes of the velocity profile with increasing distance from the entrance of the channel is shown in Fig. 3 for $Re = 30 \cdot 10^3$. It is seen that the initial increase of $\bar{x}$ decreases the filling of the velocity profile. After the disappearance of the flow core, this variation becomes opposite: with increasing distance from the entrance, the ratio U/U_0 increases over the whole cross section. At the distance $\bar{x} \approx 56$, the formation of the profile of average velocities is practically complete. The magnitude of the ratio $\bar{U}/U_0$ increases weakly with increasing Re. It increases from ≈ 0.815 at $Re = 18 \cdot 10^3$, to 0.825 at $Re = 65 \cdot 10^3$.

Even after the stabilized state has been reached, the flow did not attain a universal character with respect to Re. The investigations of the hydraulic resistance in the channels of the apparatuses GU-1 and V-1 showed that only for $Re \gtrsim 18 \cdot 10^3$ the resistance varies as

$$\lambda = 0.3164\, Re^{-0.25}. \tag{7}$$

For smaller Re, the experimental points were systematically below the curve (7). In the same region, we studied the effect of Re on the velocity profile laws (the wall law, the velocity defect law).

It is seen from Fig. 4 that for relatively small Re beyond the limits of the buffer zone, the dependence $U^+ = f(y^+)$ does not take a universal character. For Re = $4.8 \cdot 10^3$, it appears as if the velocity profile suddenly goes over to the region where

$$U^+ = A + 7.8 \lg (y^+). \tag{8}$$

As Re increases, this region is displaced towards higher values of y^+, and the magnitude of the coefficient A strongly decreases. For Re $\approx 15 \cdot 10^3 - 20 \cdot 10^3$, a zone with the known universal logarithmic variation of the velocities

$$U^+ = 6.0 + 5.5 \lg (y^+) \tag{9}$$

is formed between the buffer zone and region (8). In the central part of the flow a new, faster, increase of U^+ with y^+ is observed than that which follows from Eq. (8). Analogous theoretical and experimental results are known for pipes with a circular cross section.

NOTATION

Re = $\bar{U}d/\nu$, where $\bar{U}$ is the average flow rate in the channel, d is the side of the channel cross section, ν is the kinematic viscosity; $\bar{x} = x/d$, where x is the distance from the entrance of the channel downstream; Re_*, Reynolds number Re for which, in the cross section $\bar{x} = 50$, $\gamma = 0.5$; $U^+ = U/U_\tau$; $U_\tau = \sqrt{\tau_w/\rho}$, dynamical velocity; U, time-averaged velocity of the flow at the distance y from the wall of the channel; $y^+ = yU_\tau/\nu$; U_0, time averaged velocity of the flow at the hydraulic axis of the channel; U_0^*, maximum value of U_0 along the axis of the channel; γ, intermittence coefficient, i.e., the fraction of the total observation time during which a turbulent flow takes place at the observation point; n, average frequency of passage of turbulent locks through a given cross section; k ($\sec^{-1}$), normalizing factor of (2); and σ and $\overline{Re}$, parameters which characterize the scattering and the center of the distribution, respectively.

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